

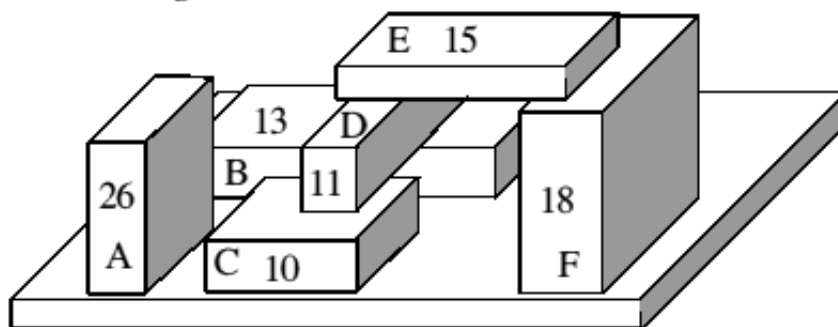
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The Scottish Mathematical Council
MATHEMATICAL CHALLENGE
 2011–2012

Solutions

Primary Division: Problems III

- P3.1.** The diagram below shows an architect's model for a new school complex. Marked on each block is the time (in months) it should take to build. The school has to be made such that no block can be started until any block on which it rests has been completed but different teams of builders were contracted to do the work so that more than one block could be built at the same time. What is the minimum construction time?
Explain your reasoning.



Solution

Block A takes 26 months
 Blocks B, D, E take 39 months
 Blocks C, D, E take 36 months
 Blocks F and E take 33 months
 The minimum time would be 39 months

- P3.2.** A water-tank can be filled by any combination of three different taps. With the smallest tap the tank can be filled in 20 minutes. With the middle tap the tank can be filled in 12 minutes. With the largest tap the tank can be filled in 5 minutes. How long does it take to fill the tank with all three taps running?
Explain your reasoning.

Solution

In 1 minute, the smallest tap fills $\frac{1}{20}$ of the tank.
 In 1 minute, the middle tap fills $\frac{1}{12}$ of the tank.
 In 1 minute, the largest tap fills $\frac{1}{5}$ of the tank.
 So in 1 minute, running together, the amount they fill is
 $\frac{1}{20} + \frac{1}{12} + \frac{1}{5} = \frac{3}{60} + \frac{5}{60} + \frac{12}{60} = \frac{20}{60} = \frac{1}{3}$ of the tank.
 Therefore, the whole water-but is filled in 3 minutes.

- P3.3.** The children were playing with the bouncing balls in the playground when I arrived there. A ball reached my full height before falling back down and I am 1.6 metres tall. “Wow! That was some bounce!”, I said, but the boy responded, “That was nothing – you should have seen the first bounce. That was the third bounce”. Each bounce of this particular ball is 20% less than its previous bounce. What height was the first bounce?

Explain your reasoning.

Solution

20% is $\frac{1}{5}$ so each bounce is $\frac{4}{5}$ of the previous one.

So if the first bounce was x metres, the second one was $\frac{4}{5}x$ metres.

So the the third bounce would have been $\frac{4}{5} \times \frac{4}{5}x = \frac{16}{25}x$ metres.

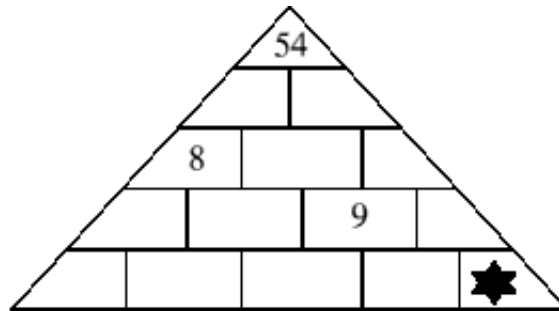
$$\frac{16}{25}x = 1.6$$

$$\frac{1}{25}x = 0.1$$

$$x = 0.1 \times 25$$

$$= 2.5$$

i.e. the first bounce was 2.5 metres.

P1.3

In each region of the triangle shown there is a whole number, three of which are given. Each number is the sum of the two numbers immediately below it and all numbers are different. Find out which number must be in the region marked with the star and explain why.

Solution

As the numbers are all different, the smallest possible numbers in the bottom row are 1, 2, 3, 4 and 5. Thus the smallest possible number in the second row is 3.

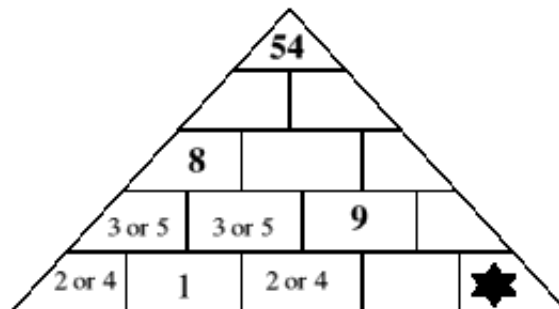
To get the 8, the cells below it must contain 3 and 5.

Below the 3, we must have 1 and 2.

Below the 5, we could have 2 and 3 or 1 and 4, but 3 cannot be repeated so 5 comes from $1 + 4$.

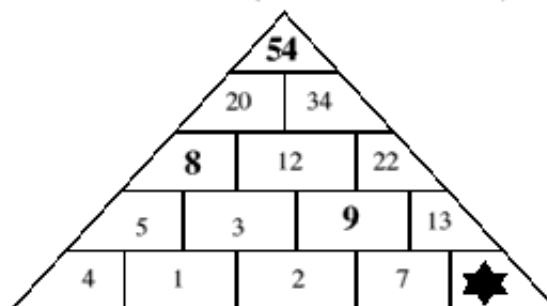
As 1 feeds into 3 and into 5, it must be directly below the 8 with 4 and 2 on either side.

So we have:



But 4 cannot feed into 9 as 5 is already used. So 4 is on the left.

This means that below 8 we have 5 and 3 (rather than 3 and 5).



The rest can now be filled in: 12, 20, 34, 13, 7 and finally the required number is 6.